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69. Proposed by GEORGE LILLEY, Ph. D., LL. D., Professor of Mathematics, University of Oregon, Eugene, Oregon.

An elliptic fence encloses a field whose major and minor axes are $2a$ and $2b$, respectively. The ends of a rope, the length of which is equal to the length of the fence, are fastened outside the fence and at the extremities of the major axis. A horse is tethered by means of a ring which slides freely on the rope. Over how much ground can he feed? What is the length of the outside border? Find these values in square feet and feet, true to six decimal places, when the area of the field is one acre and $a=2b$.

Solution by MELLEN WOODMAN HASKELL, A. M., Ph. C., Associate Professor of Mathematics, University of California, Berkeley, Cal.

The outside border will evidently be a curve parallel to the given ellipse, so that the two curves will have common normals. Let ds denote the element of arc on the ellipse, dS the element of arc on the parallel curve lying between the same normals, $d\phi$ the angle between those normals, dA the element of area bounded by those normals, by ds and dS, and p the length of the rope, then

$$dS=ds+pd\phi, \text{ and } dA=pds+\tfrac{1}{2}p^2d\phi.$$

Integrating around the ellipse, since in this case the perimeter of the ellipse $\int ds$ is equal to p, and also the complete integral $\int d\phi$ is evidently 2π, we have

$$S=p(1+2\pi) \text{ and } A=p^2(1+\pi).$$

A simple calculation then gives

$p=806.693$ feet, perimeter of ellipse;
$S=5875.293$ feet, length of outside border;
$A=2695155$ square feet, included area.

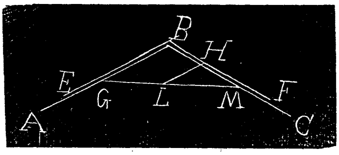
MECHANICS.

57. Proposed by J. C. NAGLE, M. A., M. C. E., Professor of Civil Engineering, Agricultural and Mechanical College of Texas, College Station, Texas.

Over the intersection of two inclined planes slides a cord of uniform mass throughout its length. Find the equation to the path described by its center of gravity.

Solution by the PROPOSER.

Let AB, BC be the inclined planes, EBF the position of the cord at any instant. Let the length of the cord be a; let BC be the x-axis, AB the y-axis. Let $EB=z$. Then the center of gravity of EB will be at G, the mid-point of EB, and of BF at M, the mid-point of BF. Let L be the center of gravity of whole cord. By moments about M,



$$GM\times z=LM\times a, \text{ or } GM/LM=a/z\text{.........}(1).$$

But from similar triangles,

$$\frac{GM}{LM}=\frac{BM}{HM}=\frac{\frac{1}{2}(a-z)}{\frac{1}{2}(a-z)-x}=a/z.$$

Whence $2az=z^2+a^2-2ax$ .... (2).

Also from the figure, $GM/LM=\frac{1}{2}z/y=a/z$, from which $z=\sqrt{(2ay)}$.........(3).

Substitute this value of z in (2), square and arrange terms, and there results,

$$4x^2-8xy+4y^2-4ax-4ay+a^2=0 \quad\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Equation (4) is that of a parabola, tangent to the axes at a distance of $\frac{1}{2}a$ from the origin.

59. Proposed by **WILLIAM HOOVER, A. M., Ph. D.,** Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

Find the radius of a sphere of given specific gravity which will rest just immersed in a fluid whose density varies as its depth.

I. Solution by **HENRY HEATON, M. Sc.,** Atlantic, Iowa; **E. L. SHERWOOD, A. M.,** Superintendent of Schools, West Point, Miss., and **W. W. LANDIS, A. M.,** Professor of Mathematics, Dickinson College, Carlisle, Pa.

Put r=radius of sphere, s_1=its specific gravity, s_2=specific gravity of the fluid at the depth of 1 foot, and a=62.5 pounds=weight of a cubic foot of water. The weight of the sphere is $\frac{4}{3}as_1\pi r^3$. The weight of the sphere must equal the weight of the fluid displaced. To find this, put x=the distance of a horizontal section of the sphere below the surface. The weight of an equivalent quantity of the fluid between two sections whose distance apart is dx is $as_2\pi(r^2-x^2)xdx$. Hence the weight of fluid displaced is

$$as_2\pi\int_0^{2r}(r^2-x^2)xdx=as_2\pi r^4=\tfrac{4}{3}as_1\pi r^3. \quad \therefore\ r=s_1/s_2.$$

II. Solution by **ALFRED HUME, C. E., D. Sc.,** Professor of Mathematics, University of Mississippi, University, Miss.; and **J. C. NAGLE, M. A., M. C. E.,** Professor of Civil Engineering, in State Agricultural and Mechanical College, College Station, Texas.

Let the origin of coördinates be taken at the lowest point of the sphere, the axis of x vertical, that of y horizontal. The equation of the section of the sphere made by the xy-plane is $y^2=2Rx-x^2$, R being the sphere's radius.

If ρ be the density of the liquid at a depth of unity, then at a distance x above the origin, the density is $\rho(2R-x)$. The weight of the displaced liquid is

$$\int_0^{2R}\pi y^2dx.\rho(2R-x)g.$$

If c be the density of the sphere, its weight is $\frac{4}{3}\pi R^3cg$.

These being equal

$$\tfrac{4}{3}cR^3=\rho\int(2Rx-x^2)(2R-x)dx,$$

the limits being $2R$ and 0, $=\frac{4}{3}\rho R^4$; $R=c/\rho$.

III. Solution by the **PROPOSER.**

Let δ, δ' be the respective densities of the sphere and of the fluid at a unit depth, and take the upper extremity of the vertical diameter of the sphere for origin, that diameter being in the axis of x.